Closing Wed: HW_7A, 7B, 7C (7.8, 8.1)
Midterm 2 is Thursday
Covers: 6.4, 6.5, 7.1-7.5, 7.7, 7.8, 8.1

Entry Task: (Directly from HW 7A/5)
Determine if the integral converges and, if so, give the value it approaches.

$$\int_{-\infty}^{\infty} 9xe^{-x^2}\,dx$$

Def'n: *Improper type 2 - infinite discontinuity*

If f(x) has a discontinuity at x = a, then

$$\int_a^b f(x)dx = \lim_{t\to a^+} \int_t^b f(x)dx$$

If f(x) has a discontinuity at x = b, then

$$\int_a^b f(x)dx = \lim_{t\to b^-} \int_a^t f(x)dx$$

If the limit exists and is finite, then we say the integral *converges*. Otherwise, we say it *diverges*.

Example:

1. $\int_0^1 \frac{1}{\sqrt{x}} dx =$

Example:

2. $\displaystyle\int_0^2 \frac{x}{x-2}\,dx =$

If f(x) has a discontinuity at x = c which is **between** a and b, then

$$\int_a^b f(x)dx = \lim_{r \to c^-} \int_a^r f(x)dx + \lim_{t \to c^+} \int_t^b f(x)dx$$

In this case, we say it *converges* only if both limits separately exist and are finite.

3. $\displaystyle\int_0^\pi \frac{1}{\cos^2(x)}\,dx =$

Limits Refresher

1. If stuck, plug in values "near" t.
2. Know your basic functions/values:

$$\lim_{t\to\infty} \frac{1}{t^a} = 0\,, \qquad \text{if } a > 0.$$

$$\lim_{t\to\infty} \frac{1}{e^{at}} = 0\,, \qquad \text{if } a > 0.$$

$$\lim_{t\to\infty} t^a = \infty\,, \qquad \text{if } a > 0.$$

$$\lim_{t\to\infty} \ln(t) = \infty.$$

$$\lim_{t\to 0^+} \ln(t) = -\infty.$$

3. For indeterminant forms, use algebra and/or L'Hopital's rule

Examples:

$$\lim_{t\to 1} \frac{t^2 + 2t - 3}{t - 1} =$$

$$\lim_{t\to\infty} \frac{\ln(t)}{t} =$$

$$\lim_{t\to\infty} t^2 e^{-3t} =$$

Aside*: A small note on* ***comparison***

Suppose you have two functions f(x) and g(x) such that **0 ≤ g(x) ≤ f(x)** for all values of x.

1. If $\int_a^\infty f(x)dx$ converges,

   then $\int_a^\infty g(x)dx$ converges.

2. If $\int_a^\infty g(x)dx$ diverges,

   then $\int_a^\infty f(x)dx$ diverges

Example:

Consider

$$\int_2^\infty \frac{1}{\sqrt[3]{x^2-1}}dx$$

We don’t have a nice way to integrate, but we can determine if this converges or diverges!

Observe $\sqrt[3]{x^2-1} < x^{\frac{2}{3}}$ (why?)

Thus, $\frac{1}{x^{2/3}} < \frac{1}{\sqrt[3]{x^2-1}}$ (why?)

So

$$\int_2^\infty \frac{1}{\sqrt[3]{x^2}}dx < \int_2^\infty \frac{1}{\sqrt[3]{x^2-1}}dx$$

Application Quick Review

1. Acceleration, velocity, distance
2. Finding Areas
3. Finding Volumes (washers or shells)
4. Average value = $\frac{1}{b-a}\int_a^b f(x)dx$
5. Work = $\int_a^b (Force)(Dist)$

(a) If f(x) = "force formula at x", then

Force = f(x), *Dist* = Δx: $\text{work} = \int_a^b f(x)dx$

(Spring, leaky bucket, …)

(b) *Chain/Cable*: k = force/length

If you label top: x = 0, then

Force = k Δx, *Dist* = x, $\text{work} = \int_a^b k\,\mathrm{x}\,dx$

(c) *Pumping*: k = force/volume

If bottom is y = 0 and top is y = b,

Force = k(Area)Δy, *Dist* = b – y

$$\text{work} = \int_a^b k(\text{Area})(b-y)dy$$

8.1 Arc Length

Goal: Given y = f(x) from x = a to x = b. We want to find the ***length*** along the curve.

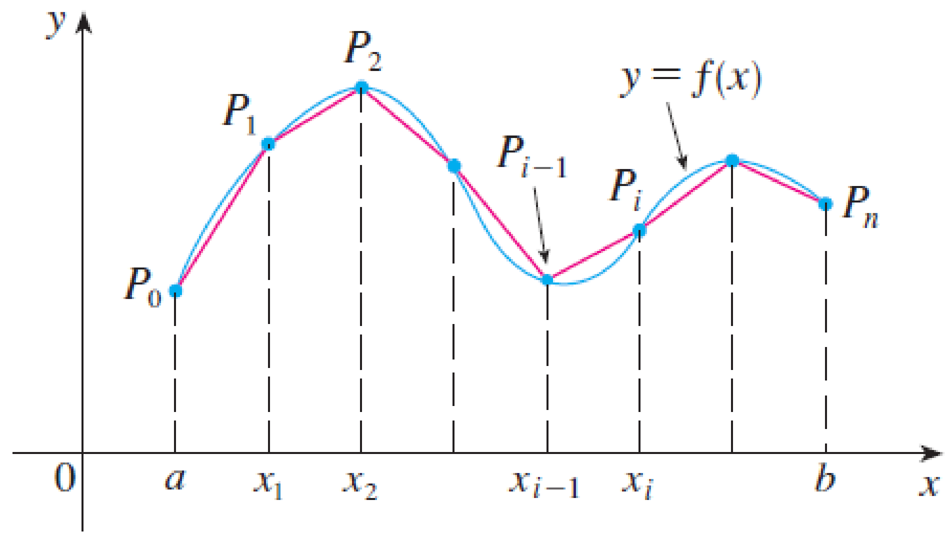
Derivation:

1. Break into *n* subdivision:
   $\Delta x = \frac{b-a}{n}, \quad x_i = a + i\Delta x$
2. Compute $y_i = f(x_i)$.
3. Compute the straight line distance from (x_i, y_i) to (x_{i+1}, y_{i+1}).

$$\sqrt{(x_{i+1}-x_i)^2+(y_{i+1}-y_i)^2} = \sqrt{(\Delta x)^2+(\Delta y_i)^2}$$
$$= \sqrt{(\Delta x)^2\left(1+\frac{(\Delta y_i)^2}{(\Delta x)^2}\right)}$$
$$= \sqrt{1+\left(\frac{\Delta y_i}{\Delta x}\right)^2}\ \Delta x$$

4. Add these distances up.

$$\text{Arc Length} = \lim_{n\to\infty}\sum_{i=1}^{n}\sqrt{1+\left(\frac{\Delta y_i}{\Delta x}\right)^2}\ \Delta x$$



Good news: We have a method to write down an integral for arc length.
Bad news: The arc length integral almost always is something that can't be done explicitly with our methods (so we have to approximate).

In the homework, you see the few, unusual cases where you actually can compute arc length explicitly.

All the 8.1 HW: Find the arc length of

1. $y = 4x - 5$ for $-3 \le x \le 2$.
2. $y = \sqrt{2 - x^2}$ for $0 \le x \le 1$.
3. $y = \dfrac{x^4}{8} + \dfrac{1}{4x^2}$ for $1 \le x \le 2$.
4. $y = \dfrac{1}{3}\sqrt{x}(x - 3)$ for $4 \le x \le 16$.
5. $y = \ln(\cos(x))$ for $0 \le x \le \pi/3$.
6. $y = \ln(1 - \mathrm{x}^2)$ for $0 \le x \le 1/7$.